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Effect of Optical Amplifier Gain on Noise in a Semiconductor Laser Amplifier for ASK-DD Technique

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Abstract : An independent, simple matlab-program based analytical approach has been demonstrated for ASK-DD optical fiber communication system with special attention to gain of semiconductor laser amplifier (SLA). This method critically evaluated noise effect on system performance when some system parameter are changed. Optical amplifier gain limit, optical input power limit and extinction ratio limit for specific optical and electric bandwidth are investigated. A novel region identification technique to differentiate between bit error rate characteristics of SLA under various conditions has been developed. The restrictions imposed by gain saturation on the bit error rate and noise

characteristics are also reported. BER close to the accepted range (around 10⁻⁹) were observed for input power -40 dBm, amplifier gain 20 dB at extinction ratio 0.04. Moreover, the hint of switching between the dominance of signal-spontaneous beat noise and spontaneous- spontaneous beat noise at points below (input power -40 dBm, amplifier gain 30 dB) and above (input power -40 dBm, amplifier gain 40 dB) the points on G vs. Pin curve is also noticeable. The results indicate that, the information gained from this research work can be useful for dynamic control of gain, extinction ratio and input power of SLA.

Keywords: Gain, extinction ratio, bit error rate, region identification, SLA, signal-spontaneous beat noise.

1. Introduction

Optical fiber based communication system has been the subject of intense research because of the (i) ever increasing traffic demand (~billions of channels to be transmitted in Tbs rate or more), (ii) fast growing development of advanced high speed opto-electronic devices [4,7] and (iii) innovative new state the art data transmission & signal processing techniques.[1,2] Bidirectional, transparent optical amplifiers hold the key for future ultrafast, extra-large transmission bandwidth optical communication systems.[9] Optical amplifiers operate completely in optical domain having flexibility to adopt any kind of multiplexing or modulation technique. In recent years, optical amplifiers have proved themselves as a promising network element (eg. linear repeaters, Inline amplifiers, optical gate pulse shapers, routing switches, detectors etc.).[4,7] Semiconductor laser amplifiers (SLA) are the most

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THE AUST JOURNAL OF SCIENCE and Technology Vol - I, Issue - I, January '09

developed generic type of all the optical amplifiers.[9] They have the single mode waveguide structure (utilize stimulated emission from the injected carriers, operation based on 'lasing' between two mirrors in the active region), exhibit low power consumption, and can be used in both linear and non-linear modes. SLA draws significant attention in high speed long haul WDM transmission because they provide high gain over wide spectral bandwidth (flat gain profile).[5,6] SLAs are classified into two main types - (i) FPA and (ii) TWA. FPA is very sensitive to fluctuation of current, temperature and polarization. Whereas, TWA with reduced reflectivity provides better performance in signal gain saturation and noise characterization. The main obstruction for the high speed reliable transmission through the SLA is the different types of noises associated with SLA- thermal noise, shot noise, signal-spontaneous beat noise and spontaneous-spontaneous beat noise.[5,6] Some noise depend on the optical amplifier gain, input power, extinction ratio etc. and damage important communication system performance parameter such as bit error rate (BER). Several studies have been performed on the system performance of SLA for basic light wave systems [5], multichannel optical networks [6], 1.5 μ m GaInAsP travelling wave SLA [8] etc. These studies are not complete to analyze BER, because comprehensive research on the following - (i) effect of extinction ratio, (ii) detail study with respect to rate equation, (iii) effect of modulation technique on various noises / bit error rate, (iv) region identification for flexible gain control [3] etc. are not yet done. We understand that a new intelligent, interactive, adaptive gain control scheme can be employed in SLA and other types of optical amplifiers to automatically vary its gain under changed input power and extinction ratio condition. In this paper, we have demonstrated a new technique to identify the 3 dimensional regions (described by extinction ratio, input power and gain as the three co-ordinate axis) of SLA on the basis of BER calculation for ASK-DD technique.

The paper is organized as follows - Section-2 devoted to theoretical background, results are reported in Section-3, discussion on results are in Section-4 and concluding remarks are made in Section-5.

2. Theoretical background

2.1 Amplifier Model

SLA consists of paired mirror (feedback instigator) and a laser medium (active region- collection of atoms or molecules of semiconductor material). Light passes through the medium, due to feedback between mirrors laser oscillation starts. A pumping source is necessary to achieve population inversion required for lasing.

We have studied a simple model with an optical amplifier (SLA in this case) followed by a photo-detector (whose spectral response is limited by a filter with bandwidth 'Be' (Fig.2.1). [5,9] The ASK input to the amplifier consists of On-Off modulated signal & photo-detector's detection is basically a photon counting process where each detected photon is converted into an electron-hole pair. The various parameters to be considered are summarized in the Table 2.1. [5]

2.2 Rate equation

Given the relationship between G and P_{in} and using the input statistics, previous researchers have computed the statistics of the gain and consequently the output for a TWA type SLA with negligible residual facet reflectivity. When the data rate is smaller than the reciprocal of the lifetime, the ion density attains steady state within a small fraction of bit period. [6]



Fig. 2.1 Block diagram of the full ASK-DD system including the amplifier model inside the dotted portion. [9] Here, I_{ip} =information input at transmitter, I_{op} =information output at receiver.

In the subsections 2.2-2.4, several equations about steady state charge conservation, noises, BER are introduced. ^[5,6] These equations are used in our study to understand the characteristics of various noises, BER as a function of *G*, *P*_{in} and *r*. The rate equation for steady state charge conservation is solved to yield the *G* as a function of *P*_{in} ^[5,6]

$$P_{in} = (P_{sat} / (G - 1)) ln(G 0 / G)$$
 (2.1)

THE AUST . JOURNAL OF Science and Technology Vol - I, Issue - I, January '09

Where, P_{sat} = internal-saturation optical power (typical =-6 dBm); the signal gain saturation occurs due to the decrease in the amount of population inversion induced by the increase in the stimulated emission.^[8] Go= maximum amplifier gain when input power is zero (typical Go=1800). G is a monotonically decreasing function of P_{in} (Fig. 2.2 -reproduced from the exact one).

Parameter	Meaning	Parameter	Meaning
Be	Electrical bandwidth	η ir η out	Amp. input & output coupling
Bo	Optical bandwidth		efficiency
e	Electron charge	Nsp	Spontaneous emission factor
hv	Photon energy	Nshot	Shot noise
Isp	PCE of sp. emission power	Ns-sp	Signal-spont. beat noise
Is(1), Is(0)	PCE of amplifier input for Mark	Nsp-sp	Spontspont beat noise
	& Space	Nth	Thermal noise
L	Optical loss between amplifier	Pin	Amplifier input power
	and receiver	P_{sp}	Spont. emission power
r	Extinction ratio	C . C	Elec. Signal power for Mark
G	Optical amplifier gain	S(1), S(0)	& Space

Table 2.1 Definitions	of Symbol	used ^[6]
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Fig.2.2 Amplifier gain vs. Input power for a typical $1.5\mu m$ SLA (figure reproduced) ^[6]

2.3 Extinction ratio (r)

The ratio of optical power in '1' state to that in the '0'-state for ASK-DD technique is termed as extinction ratio. In non ideal cases (when optical sources are prebiased/ not fully off during '0'-state) - some power is emitted during '0' pulse. [6,9]

Mathematically,

$$r = P_0 / P_1 \tag{2.2}$$

(where, P1=power transmitted for a one bit, Po=power transmitted for a zero bit) [6]

$$P_{1} = 2P_{in} / (1+r) \tag{2.3a}$$

$$P_0 = 2_{\rm r} P_{in} / (1+r) \tag{2.3b}$$

2.4 Mathematical expression of Bit error rate

Photo-current equivalent of spontaneous emission power,

$$I_{sp} = (P_{spe}/hv) = N_{sp} (G-1) eB_0$$
(2.4)

Where, spontaneous emission power at output from SLA,

$$P_{sp} = N_{sp}(G-1) hv \tag{2.5}$$

After square law detection in the receiver, the received signal power,

$$S = (GIs\eta in \eta outL)^2$$
(2.6)

For an amplitude modulated signal of average power P_{in} 50% duty cycle and an extinction ratio 'r', the photo current equivalent of input powers for a mark I_s (1) and space $I_s(0)$ are

$$I_{s}(1) = eP_{in}2r/(hv(r+1))$$
(2.7a)

&
$$I_s(0) = eP_{in}2/(hv(r+1))$$
 (2.7b)

The bit error rate is given by,

$$BER = (1/\sqrt{2\pi})(exp(-Q^{2/2})/Q)$$
(2.8a)

Where
$$Q = [\checkmark S(1) - \checkmark S(0)] / [\checkmark Ntot(1) + \checkmark Ntot(0)]$$
 (2.8b)

Total noise

$$N_{tot}(1) = N_{th} + N_{s-sp}(1) + N_{sp-sp} + N_{shot}(1)$$

$$(2.8c)$$

$$N_{tot}(0) = N_{th} + N_{s-sp}(0) + N_{sp-sp} + N_{shot}(0)$$

$$(2.8d)$$

Mathematical expression for various noises

$$Nth = Ith^2 Be \tag{2.8e}$$

$$N_{s-sp}(1) = [4GI_s(1) \eta_{in} \eta_{out}^2 I_{sp} L^2 B_e]/B_o$$
(2.8f)

$$N_{sp-sp} = [(I_{sp}\eta_{out}L)^2 Be(2B_0 - B_e)] / B_0^2$$
 (2.8g)

$$N_{shot}(1) = 2eBe\eta_{out}L[GI_{s}(1)\eta_{in} + I_{sp}]$$

$$(2.8h)$$

3. Results :

3.1 The effect of optical amplifier gain on Bit error rate (Region identification)

We have observed the effect of optical amplifier gain on bit error rate for various extinction ratio (r = 0.04, 0.25, 0.5, 0.75 and 0.90) for different input power (P_{in} = -40 dBm, -45dBm etc.). Some fixed values were chosen for N_{sp} , B_o , B_e etc. We have classified the BER into four categories (BER <10⁻²⁰ as 'G', 10⁻²⁰ <BER<10⁻³ as 'R', 10⁻³<BER<0 as 'B' and BER=0 as 'Z'). BER =10⁻⁹ is always an important and accepted rate for data transmission through an optical device, so in order to understand the nature of various noises near bit error rate=10⁻⁹; we have categorized 10⁻²⁰ <BER<10⁻³ as the region of research interest and termed it as 'R' region. The various regions are tabulated for various input power conditions. In Table 3.1, summary of result for two cases (P_{in} =-30 dBm & P_{in} =-40 dbm) is shown. Similar studies also have been performed for other input power (0 to -50 dBm). The nature of the result is reported in the following paragraph.

		$P_{in} = -30$	0 dBm	 $P_{in} = -40 \text{ dBm}$					
r \G	0-10	10-20	20-30	30-50	0-10	10-20	20-30	30-50	
0.04	R	G	Z	Z	В	R	G	G	
0.25	В	G	Ζ	Z	В	В	G	G	
0.5	В	G	G	G	В	В	R	R	
0.75	В	R	G	G	В	В	R	R	
0.9	В	В	R	R	В	В	В	В	

Table 3.1 Region identification for $P_{in} = -30 \text{ dBm } \& -40 \text{ dBm}$

3.1.1 (Nature of result- gain region '20-50 dB')

(i) When input power is average (P_{in} =-40 to -20 dBm) - It is evident that, various regions transform from one type to another when 'r' and ' P_{in} ' are changed (Table 3.2). 'Z' region was observed for low/medium r and high P_{in} , 'B' for high r- low P_{in} , 'G' for low r-low P_{in} / high r-high P_{in} and 'R' for high r-low & medium P_{in} , For a particular r when P_{in} is increased, the region transformed in the $B \rightarrow R \rightarrow G \rightarrow Z$ direction. For a particular P_{in} , when r increases, the region transformation is in $Z \rightarrow G \rightarrow R \rightarrow B$ direction. Some critical transitions between regions are indicated by arrows.

$r \setminus P$ in	-20 dBm	-25 dBm	-30 dBm	-35 dBm	-40 dBm
0.04	Z	Z	Z	$Z \rightarrow \downarrow$	G
0.25	Z	Z	$Z \rightarrow$	G	G↓
0.5	Z↓	$Z \rightarrow \downarrow$	G	$G \rightarrow \downarrow$	R
0.75	G	G	$G \rightarrow \downarrow$	R↓	R↓
0.9	$G \rightarrow$	R	$R \rightarrow$	В	В

Table 3.2	Nature	of result	(Gain	20-50	dB)
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(*ii*) When input power is low ($P_{in} < -40 \ dBm$) The pattern remains the same when we have increased 'r' and ' $P_{in'}$. 'R'-region was observed for some specific combination of 'r', P_{in} and G values (summarized in Table 3.3).

(iii) When input power is high $(P_{in} > -20 \ dBm)$ - There is no 'R'-region. 'Z' region for $P_{in}=0$ ~-10 dBm (all 'r'), 'G' region for r = 0.9 ($P_{in}=-15 \ dBm$), and 'Z' region for r=0.04-0.75 ($P_{in}=-15 \ dBm$) is observed.

Table 3.3	'R'-	region	identification	for	Gain	20-50	dB	&	Pin	<	-40	dBm
		0										

G= 20-30 dB	G=30-50 dB						
1. <i>Pin</i> =-45 dBm, <i>r</i> =0.04-0.5	1. P_{in} =-45 dBm, $r = 0.25 - 0.5$						
2. P_{in} =-50 dBm, r =0.04	2. P_{in} =-50 dBm, r =0.04-0.5						

3.1.2. (Nature of result- 'gain region, G <20 dB')

When input power is average (P_{in} =-40 to -20 dBm) - 'R'-region was observed for high P_{in} & small *r* combination & vice versa. Nature remains the same. So, if we increase the gain, then the possibility of detecting 'R'-region become high for lower P_{in} and *r* (Table 3.4). When input power is low (P_{in} <-40 dBm) - low P_{in} and *G* is a bad combination. When input power is high ($P_{in} > -20$ dBm) -'R'-region was observed only for P_{in} =-15 dBm, *r*=0.9 & G=0-10 dB condition. The rest are 'G' and 'Z' region.

	-40 dBm	-35 dBm	-30 dBm	-25 dBm	-20 dBm
0-10 dB			r=0.04	r=0.25-0.5	r=0.75
10-20 dB	r=0.04	r=0.25-0.5	r=0.75	r=0.9	

Table 3.4 'R'- region identification for Gain= 0-20 dB & Pin =-40 ~ -20 dBm

In Table 3.5, we have summarized the overall results. The observation were (i) r = 0.9, Pin < -40 dBm all 'G' to 'B', (ii) low and medium Pin -transition B \rightarrow R occurs for G>20 dB and (iii) High Pin, transition occurs when G> 10 dB.

$\mathbf{r} \setminus P$ in	-50	dBm	-40 dBm			-	-30 dB	m	-20 dBm		
0.04	В	R	В	R	G	R	G	Ζ	G	Z	
Gain	0-20	20-50	0-10	10-20	20-50	0-10	10-20	20-50	0-10	10-50	
0.25	В	R	В		G	В	G	Ζ	G	Z	
Gain	0-30	30-50	0-20) 20)-50	0-10	10-20	20-50	0-10	10-50	
0.50	В	R	В		R	В		G	G	Ζ	
Gain	0-30	30-50	0-2	0 20)-50	0-1	0 1	0-50	0-10	10-50	
0.75		В	В		R	В	R	Ζ	R	G	
Gain	2	all	0-20 20-50		0-10	10-20	20-50	0-10	10-50		
0.90		В	В		I	3	R	В	G		
Gain	8	all		all		0-2	20 20	0-50	0-10	10-50	

Table 3.5 Overall summary of Region transition (*Pin* = -20 ~ -50 dBm)

3.2 The effect of optical amplifier gain on Bit error rate (graphical nature)

The graphical relationship between BER and Gain in the G, R, Z and B region is shown in Fig.3.1(a)-(d) respectively. [Figure should be considered in clockwise (cw) direction for (a)-(d). *Pin* for (a) -45 dBm, (c) -35 dBm, (b) & (d) -50 dBm.; r = 0.04] Graphs for 'G' and 'R' region can be divided into three parts (i) higher slope fall, (ii) lesser slope fall and (iii) flat region. BER vs. Gain curve in 'R'-region is shown in Fig.3.2. *Pin* for (a), (b) -50 dBm, (c) -45 dBm & (d) -35 dBm.; r = 0.04].



Effect of Optical Amplifier Gain on Noise in a Semiconductor Laser Amplifier for ASK-DD Technique

Fig. 3.1 BER vs. Gain curve in 'G', 'R', 'Z' and 'B'-region, [(a)-(d) cw direction]





3.3 The effect of optical amplifier gain on various noises (graphical nature)

Our next study was to observe the effect of optical amplifier gain on four different types of noises as a function of extinction ratio & power input. Figure 3.3-3.5 shows representative curves for N_{spsp}, N_{sbp}, N_{sbot} respectively for r = 0.04. All of them have shown similar continuously increasing nature with gain. On the other hand, both N_{ssp} and N_{sbot} has increased with *Pin* (In Fig. 3.4-3.5, all the bottom/lowest valued graphs are for *Pin*=-50 dBm, highest one for -40 dBm; -48 dBm and -45 dBm in between)



Fig. 3.3 Spontaneous-spontaneous beat noise vs. Gain curve [Pin=-50 dBm, r=0.04]



Fig. 3.4 Signal-spontaneous beat noise vs. Gain curve [Pin=-50~-40 dBm, r=0.04]



Effect of Optical Amplifier Gain on Noise in a Semiconductor Laser Amplifier for ASK-DD Technique

Fig. 3.5 Shot noise vs. Gain curve [*Pin*=-50 (lowest), -48, -45,-40 dBm, r=0.04]

4. Discussion on results:

4.1 On the basis of gain saturation/ rate equation point of view

a) According to Table 3.5, several transitions occur near [G, Pin] = [30,-40], [20,-30], [10,-20] co-ordinates. These Pin values correspond to linear portion of G vs. Pin graph (Fig.2.1). In the saturation portion, normally 'B'-region was observed. But, with the increase in gain for low 'r' values, the region transform into 'R' (like the linear portion of G vs. Pin graph). Moreover, in the linear portion of Fig. 2.1, no 'B' region was observed (indication of satisfactory BER). So, the close relation-ship between G vs. Pin graph and region identification table is observed.

b) According to Fig. 3.1-3.2, the nature of gain dependence of BER graphs (near linear portion of Fig. 2.1, i.e. $G = 20-30 \text{ dB} \& P_{in}=-40, -50 \text{ dBm}$) remains the same. But, near saturation region ($P_{in}=-45,-50 \& G=30-40$), the characteristics is different. So, the effect of gain saturation on BER vs. Gain graph is clearly visible.

4.2 On the basis of noise- point of view

Signal-spontaneous beat noise (N_{ssp}), Spontaneous-spontaneous beat noise (N_{spsp}), Shot noise (N_{shot}) are closely studied in the present work. Practically, N_{ssp} is unavoidable because it arises from the beat between amplifying signal power and ASE (accu-

THE AUST JOURNAI OF Science and Technology Vol - I, Issue - I, January '09

mulated stimulated emission) noise power around signal frequency. With the increase of amplifier gain, N_{ssp} has increased in a non-linear fashion (Fig. 3.4). For a particular gain, N_{ssp} has also increased with the increase of signal power (Fig. 3.4 (a)-(d), the bottom graph is for P_{in} =- 50 dBm and top one for P_{in} =-40 dBm; -48 and -45 in between). The presence of proportional G^2 and P_{in} term in equation (2.8f) has contributed to this kind of result. According to Table 3.1, these graphs correspond to 'Z' region (with more than acceptable bit error rate). If we relate these graphs with Fig. 2.1, we observe that [32.5, -50] point corresponds to saturation and [32,-40] point corresponds to transition between saturation & linear portion of G- P_{in} curve. Now, when gain is changed between 30 and 32.5 dB (means towards the saturation region point), the N_{ssp} vs. G curves for several P_{in} are close to each other. Above 32.5 dB (means going away from the saturation region point), the slope of the curve becomes more for higher P_{in} values. It indicates amplifier is affected more by P_{in} above the actual point on the G- P_{in} curve in saturation region than that below. Similar behavior was also observed for the transition point on the G- P_{in} curve.

On the other hand, N_{spsp} arises from ASE beat noises over wide frequency (10 GHz in this study). According to eq. 2.8g, N_{spsp} is only a function of 'G' and not depend on P_{in} . So, variation was observed for G only. According to Figure 3.3-3.5, near saturation region point [30, -50], $N_{spsp} > N_{ssp} > N_{shot}$ (N_{spsp} dominates). If we incresase gain, then above the exact point on G- P_{in} curve, at [40,-50], $N_{ssp} > N_{shot}$ (N_{ssp} dominates). Similar result was also observed for point below ([30,-40]) and above ([40,-40]) the transition point on G- P_{in} curve. N_{shot} is the lowest one of them because two beat noises are mathematically G-times larger than N_{shot} . If we compare these graphs with Table 3.1, for points above G- P_{in} curve, the transition occurs between 'G' and 'R' region could be the result of this switching between the dominance of N_{ssp} and N_{spsp} .

5. Conclusions :

The results of the present work can be summarized as follows :

- (a) For the first time, a comprehensive study has been done on the effect of mutual interrelationship between optical amplifier gain, input power and extinction ratio on the BER performance of SLA.
- (b) Successfully, a new region identification study is made in three dimensions (gain, input power and extinction ratio) on the basis of BER calculation.
- (c) A study has been done on the effect of optical amplifier gain on various noises-(signal spontaneous beat noise etc.) and consequent effect on BER.
- (d) The results are successfully explained by the inherent nature of SLA such as

gain saturation, noise dominance etc. Hint of switching between the dominance of two beat noises above and below points on the G- P_{in} curve is also observed for the first time.

(e) Author realizes that, the result of this study is invaluable for real-time dynamic gain control for SLA and other types of optical amplifiers.

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